

Chose one of the following subjects to solve using matlab

## Subject #1: heat transfer

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**Problem #1:** the following equation is for heat conduction:

$$\frac{d^2T}{dy^2} = -\frac{\dot{q}}{k}$$

If:

q is the heat transfer in y-direction= 1000 Watt/m<sup>3</sup>

k is the thermal conduction coefficient = 40 W/m.K

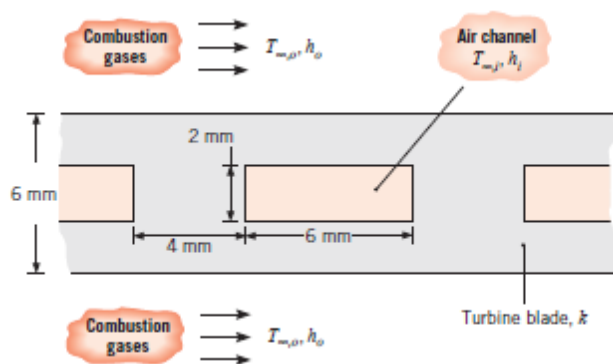
T is the temperature; in Kelvin

**Write a Matlab program to solve this problem if:**

1. y-range : 0 – 0.5m
2. T(0)=20 K;
3. dT/dy(0)=1 K/m.

**Problem #2:** Write a Matlab program to solve the following problem

A major objective in advancing gas turbine engine technologies is to increase the temperature limit associated with operation of the gas turbine blades. This limit determines the permissible turbine gas inlet temperature, which, in turn, strongly influences overall system performance. In addition to fabricating turbine blades from special, high-temperature, high-strength superalloys, it is common to use internal cooling by machining flow channels within the blades and routing air through the channels. We wish to assess the effect of such a scheme by approximating the blade as a rectangular solid in which rectangular channels are machined. The blade, which has a thermal conductivity of  $k = 25 \text{ W/m}\cdot\text{K}$ , is 6 mm thick, and each channel has a  $2 \text{ mm} \times 6 \text{ mm}$  rectangular cross section, with a 4-mm spacing between adjoining channels.



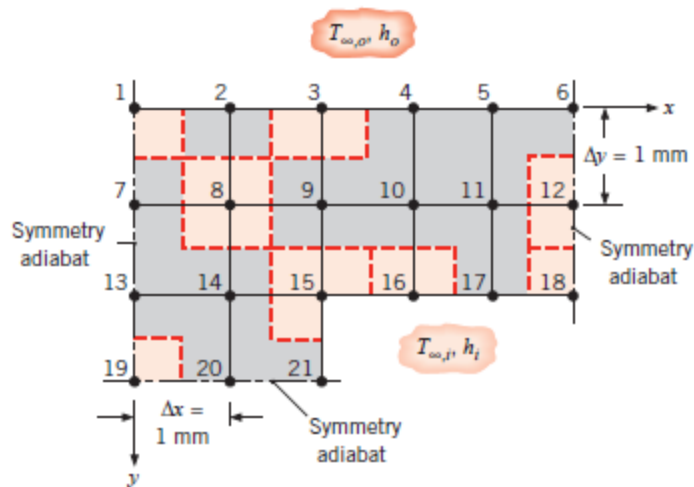
Under operating conditions for which  $h_o = 1000 \text{ W/m}^2 \cdot \text{K}$ ,  $T_{\infty,o} = 1700 \text{ K}$ ,  $h_i = 200 \text{ W/m}^2 \cdot \text{K}$ , and  $T_{\infty,i} = 400 \text{ K}$ , determine the temperature field in the turbine blade and the rate of heat transfer per unit length to the channel.

**SOLUTION**

**Known:** Dimensions and operating conditions for a gas turbine blade with embedded channels.

**Find:** Temperature field in the blade, including a location of maximum temperature, of heat transfer per unit length to the channel.

**Schematic:**



**Assumptions:**

1. Steady-state, two-dimensional conduction.
2. Constant properties.

**Analysis:** Adopting a grid space of  $\Delta x = \Delta y = 1$  mm and identifying the three lines of symmetry, the foregoing nodal network is constructed. The corresponding finite-difference equations may be obtained by applying the energy balance method to nodes 1, 6, 18, 19, and 21 and by using the results of Table 4.2 for the remaining nodes.

Heat transfer to node 1 occurs by conduction from nodes 2 and 7, as well as by convection from the outer fluid. Since there is no heat transfer from the region beyond the symmetry adiabat, application of an energy balance to the one-quarter section associated with node 1 yields a finite-difference equation of the form

$$\text{Node 1: } T_2 + T_7 - \left(2 + \frac{h_o \Delta x}{k}\right) T_1 = -\frac{h_o \Delta x}{k} T_{\infty,o}$$

A similar result may be obtained for nodal region 6, which is characterized by equivalent surface conditions (2 conduction, 1 convection, 1 adiabatic). Nodes 2 to 5 correspond to case 3 of Table 4.2, and choosing node 3 as an example, it follows that

$$\text{Node 3: } T_2 + T_4 + 2T_9 - 2\left(\frac{h_o \Delta x}{k} + 2\right) T_3 = -\frac{2h_o \Delta x}{k} T_{\infty,o}$$

Nodes 7, 12, 13, and 20 correspond to case 5 of Table 4.2, with  $q'' = 0$ , and choosing node 12 as an example, it follows that

$$\text{Node 12: } T_6 + 2T_{11} + T_{18} - 4T_{12} = 0$$

Nodes 8 to 11 and 14 are interior nodes (case 1), in which case the finite-difference equation for node 8 is

$$\text{Node 8: } T_2 + T_7 + T_9 + T_{14} - 4T_8 = 0$$

Node 15 is an internal corner (case 2) for which

$$\text{Node 15: } 2T_9 + 2T_{14} + T_{16} + T_{21} - 2\left(3 + \frac{h_i \Delta x}{k}\right) T_{15} = -2\frac{h_i \Delta x}{k} T_{\infty,i}$$

while nodes 16 and 17 are situated on a plane surface with convection (case 3):

$$\text{Node 16: } 2T_{10} + T_{15} + T_{17} - 2\left(\frac{h_i \Delta x}{k} + 2\right) T_{16} = -\frac{2h_i \Delta x}{k} T_{\infty,i}$$

In each case, heat transfer to nodal regions 18 and 21 is characterized by conduction from two adjoining nodes and convection from the internal flow, with no heat transfer occurring from an adjoining adiabat. Performing an energy balance for nodal region 18, it follows that

$$\text{Node 18: } T_{12} + T_{17} - \left(2 + \frac{h_i \Delta x}{k}\right) T_{18} = -\frac{h_i \Delta x}{k} T_{\infty, i}$$

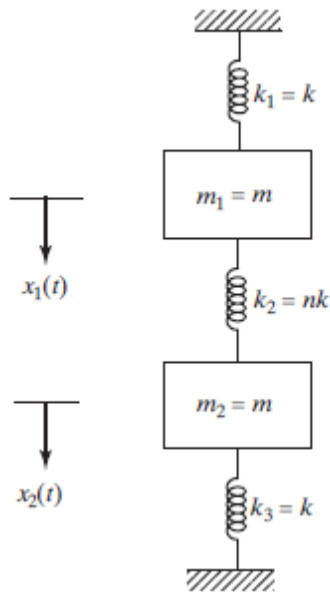
The last special case corresponds to nodal region 19, which has two adiabatic surfaces and experiences heat transfer by conduction across the other two surfaces.

$$\text{Node 19: } T_{13} + T_{20} - 2T_{19} = 0$$

# Subject #2: mechanical vibrations and strength of materials

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**Problem#3:** for the following 2-degree of freedom vibration system shown in the figure



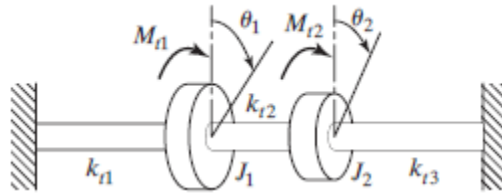
The governing equations are:

$$\begin{aligned} m\ddot{x}_1 + 2kx_1 - kx_2 &= 0 \\ m\ddot{x}_2 - kx_1 + 2kx_2 &= 0 \end{aligned}$$

Write a Matlab program to find  $x_1$  and  $x_2$  if:

1.  $m=1$  kg
2.  $k=1000$ N/m
3.  $x_1(0)=x_2(0)=0$
4.  $dx_1/dt(0)=0$  &  $dx_2/dt(0)=0$

**Problem#4:** for the following 2-degree of freedom vibration system shown in the figure



The governing equations are:

$$J_1 \ddot{\theta}_1 + (k_{r1} + k_{r2})\theta_1 - k_{r2}\theta_2 = M_{r1}$$

$$J_2 \ddot{\theta}_2 - k_{r2}\theta_1 + (k_{r2} + k_{r3})\theta_2 = M_{r2}$$

Write a Matlab program to find  $x_1$  and  $x_2$  if:

$$k_{r1} = \frac{GI_{01}}{l_1} = \frac{G}{l_1} \left( \frac{\pi d_1^4}{32} \right) = \frac{(80 \times 10^9)(\pi)(0.10)^4}{(0.8)(32)} = 981,750.0 \text{ N-m/rad}$$

$$k_{r2} = \frac{GI_{02}}{l_2} = \frac{G}{l_2} \left( \frac{\pi d_2^4}{32} \right) = \frac{(80 \times 10^9)(\pi)(0.15)^4}{(1.0)(32)} = 3,976,087.5 \text{ N-m/rad}$$

$$M_{r1} = M_{r2} = 2 \text{ kN.m}$$

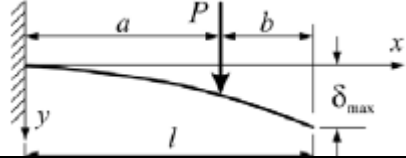
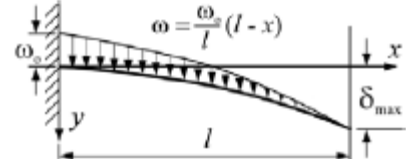
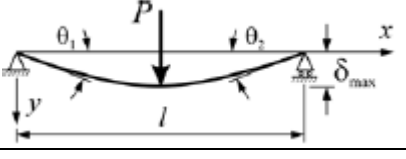
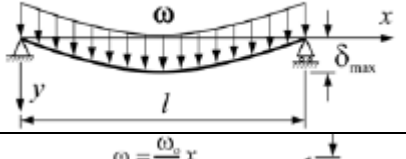
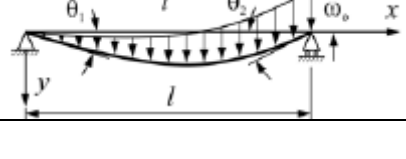
$$J_1 = J_2 = 600 \text{ kg-m}^2$$

**Problem #5:** for the damped system described by the following equation

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$


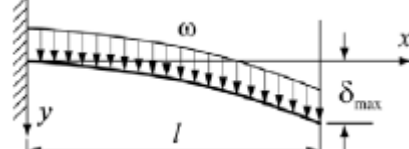

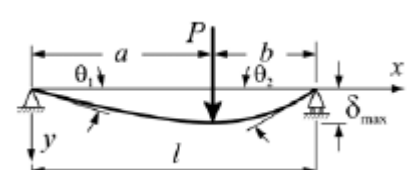
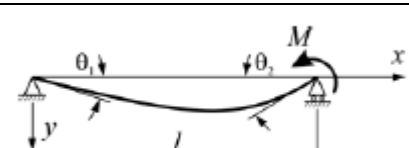
Solve to find  $x(t)$  using Matlab. Assume zero I.Cs

**Problem #6:** the following are beam deflection equations. Draw the beam deflection curve using matlab for a chosen beam from your own selection.

Case	System description	DEFLECTION AT ANY SECTION IN TERMS OF $x$
1		$y = \frac{Px^2}{6EI}(3a-x) \text{ for } 0 < x < a$ $y = \frac{Pa^2}{6EI}(3x-a) \text{ for } a < x < l$
2		$y = \frac{\omega_0 x^2}{120EI}(10l^3 - 10l^2x + 5lx^2 - x^3)$
3		$y = \frac{Px}{12EI} \left( \frac{3l^2}{4} - x^2 \right) \text{ for } 0 < x < \frac{l}{2}$
4		$y = \frac{\omega x}{24EI}(l^3 - 2lx^2 + x^3)$
5		$y = \frac{\omega_0 x}{360lEI}(7l^4 - 10l^2x^2 + 3x^4)$

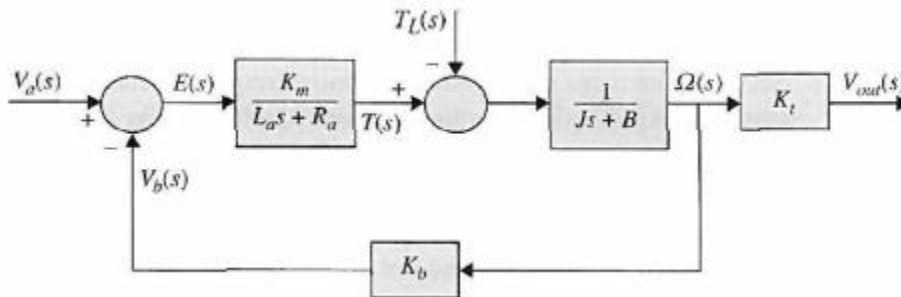


**Problem #7:** the following are beam deflection equations. Draw the beam deflection curve using matlab for a chosen beam from your own selection.

Case	System description	DEFLECTION AT ANY SECTION IN TERMS OF $x$
1		$y = \frac{Px^2}{6EI}(3l - x)$
2		$y = \frac{\omega x^2}{24EI}(x^2 + 6l^2 - 4lx)$
3		$y = \frac{Mx^2}{2EI}$
4		$y = \frac{Pbx}{6EI}(l^2 - x^2 - b^2) \text{ for } 0 < x < a$ $y = \frac{Pb}{6EI} \left[ \frac{l}{b}(x-a)^3 + (l^2 - b^2)x - x^3 \right]$ for $a < x < l$
5		$y = \frac{Mlx}{6EI} \left( 1 - \frac{x^2}{l^2} \right)$

# Subject #3: automatic control system

**Problem#8:** use matlab simulink to simulate the following control system(find  $v(t)$ )

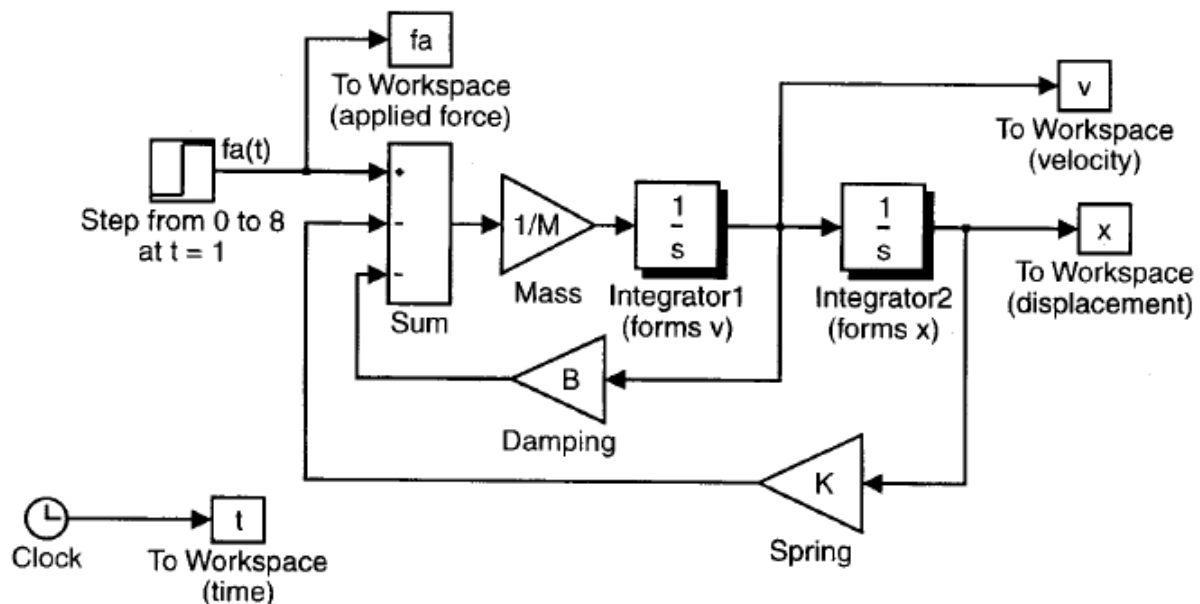


Assume

$L_a=0.003$     $R_a=5$     $K_m=1$     $T_L=0$     $J=0.0002$     $B_t=0.015$     $K_t=2$     $K_b=0.5$

Also assume step input:  $v_{in} = 5$ volt

**Problem#9:** the following simulink diagram is used to simulate a vibrating system



Redraw this system and find the response  $x$  and  $v$  if : input force that increases from 0 to 8 N at  $t=1$  s. The parameter values are  $M = 2$ kg,  $K= 16$ N/m and  $B=4$ N.s/m

# Subject #4: numerical methods

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**Problem #10:** write a matlab m-file to find the roots of the following equations using:

1. bisection method
2. false position method

the equations are:

1.  $f(x) = 5x^3 - 5x^2 + 6x - 2$
2.  $f(x) = \sin(x^2) + \cos(x)$  for  $0 < x < \pi$

**Problem #11:** write a matlab m-file to find the following integrations numerically using Simpson's 3/8 rule. use 5 iterations

$$\int_{-2}^4 \left( \frac{1x - 4x^3}{\ln(x+1)} \right) dx$$

**Problem #12:** find the solution for the following equations using matlab ode45 code where  $y=f(t)$

$$\frac{dy}{dt} = \sin(50t) - 2t^2 + 5$$
$$5y'' + 3y' + 1 = 0$$
$$t^2 y + y'' = 10$$